# RANDOM THOUGHTS ON SOCIAL NETS

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## Abstract

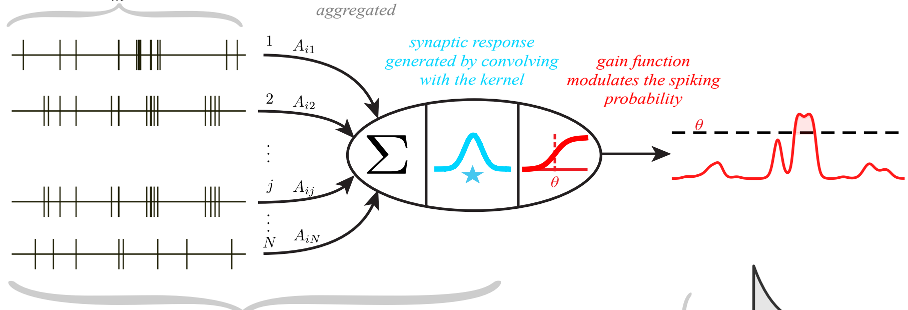
Here I have informally discussed a few ideas on modeling social force phenomena (as stated by Deb Roy) through a simplistic rate-based neural network model. My goal is to address the social *F=MA* law in a more rigorous way and point out some of its inefficiencies. In doing so, I have introduced a simple visualization method as a thinking tool for Milgram (crowd gaze) experiment. This visualization is capable of demonstrating some of the complex behaviors of social networks such as competition bias and attention inversion. At the end, I have presented a modified model that takes such behaviors into account.


## Rate-Based Neural Networks

In rate based neural networks (RBNNs) information is coded by the rate at which spikes arive to a neuron. Each spike, triggers a synaptic response that lasts for a while as though the neuron remembers the spike for a limited time. Thus, a higher rate of spikes accumulates more activity, resulting in a higher probability of spiking in the host neuron. To mathematically address these behaviors, take the schematic neuron as our model system.

$$C_j(t) = \sum_m \delta(t - T_j^m)$$

*incoming signals get weighted by the adjacency matrix and aggregated*



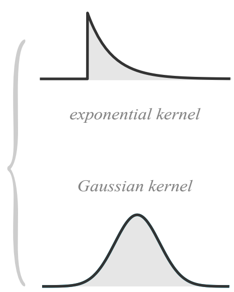

To model the synaptic response (memory kernel), we need a convolution kernel. The most natural choices that come up in different applications are Gaussian and exponential kernels. The input activity of each neuron in the network would be the convolution of the kernel with the spike-train that comes from other neuron in the network, weighted by the adjacency matrix.

$$u_i(t) = \sum_{j=1}^{N} A_{ij}\phi_{ij}(t) \star C_j(t)$$

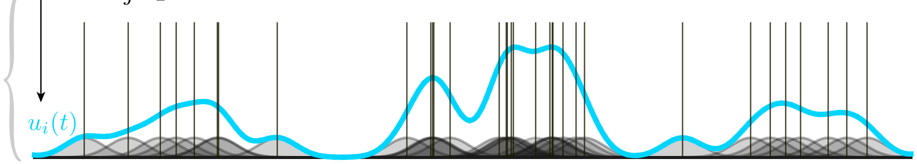
*(i,j) component of the adjacency matrix*

$$= \sum_{j=1}^{N} A_{ij} \int_{-\infty}^{t} \phi_{ij}(t-t')C_j(t')\mathrm{d}t'$$

$\phi_{ij}(t)$




In this figure, I have used a Gaussian kernel. The width of the kernel indicates the time that takes for the memory to fade. Here the blue line shows the accumulated input signal over time.

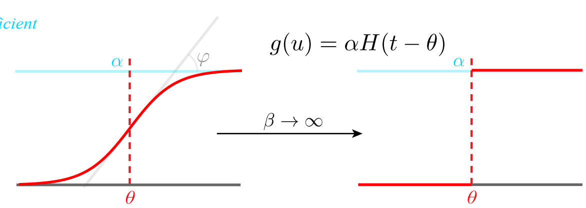



The aggregated signal passes through a gain function which modulates the probability (or the rate) of spiking. The sigmoidal and Heaviside step-function are the usual choices as they lead to exactly solvable models.

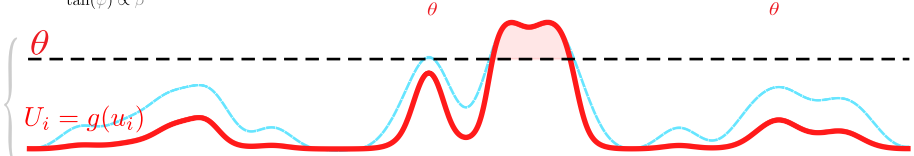
$$g(u) = \frac{\alpha}{1 + e^{-\beta(u-\theta)}}$$

*the gain coefficient* ($\alpha$) *slope of transition* $\tan(\varphi) \propto \beta$ *threshold* ($\theta$)

$$g(u) = \alpha H(t - \theta)$$




I have applied a sigmoidal gain function here and the resulting signal is shown by the red line. The dashed line shows the threshold of the gain function.




Assuming an exponential decay for the synaptic kernel and a few other technical assumptions (see section 2.4 in **Bressloff(2012)** ) can lead to a simple set of differential equations for the feedback dynamics of the network.

$$\tau_i \frac{\mathrm{d}\phi_i(t)}{\mathrm{d}t} + \phi_i(t) = \delta(t)$$

*the solution is an exponential function :* $\phi = e^{-t/\tau}H(t)$

$$U_i(t) = \sum_{j=1}^{N} A_{ij} \int_{-\infty}^{t} \phi_{ij}(t-t')\tilde{g}(U_i(t'))\mathrm{d}t'$$

*the gain function is assumed to be sigmoidal for the equations to be exactly solvable*

In this equation the synaptic kernel is modeled as its associated time-constant and the external stimulation is also included.

$$\tau_i \frac{\mathrm{d}U_i(t)}{\mathrm{d}t} = -U_i(t) + \sum_{j=1}^{N} A_{ij}\tilde{g}(U_j(t)) + S_i(t)$$

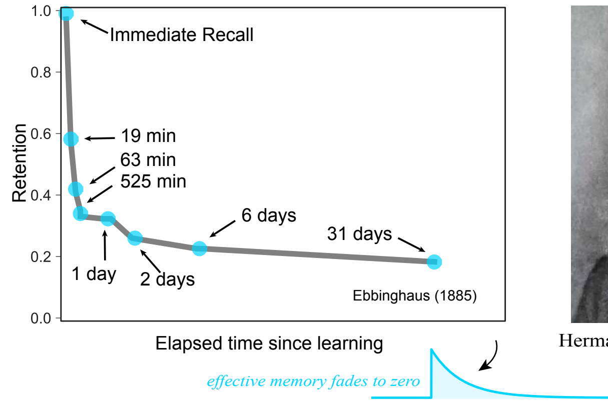
$S_i(t)$ *stimulation at i'th neuron*

## Memory Fades Fast

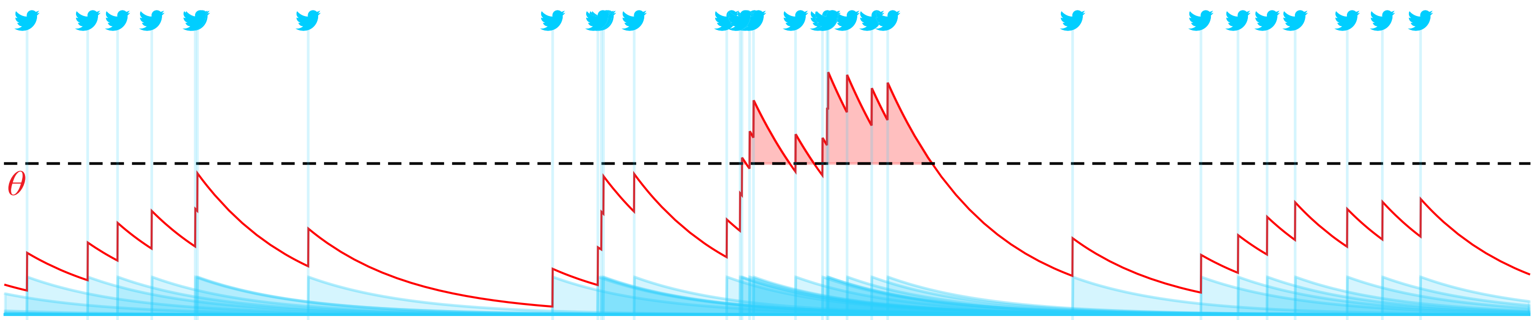
To use the RBNN model for social nets, we have to estimate the corresponding synaptic kernel and gain function.

We are going to use a slightly modify version of Ebbinghaus's results as our memory (synaptic) kernel. The problem with Ebbinghaus's curve is that it does not decay to zero. Although this is true for humans' memory, but we consider a kind of *effective memory* that does not last for ever. Therefore we will use a simple exponential kernel as defined in the previous section.



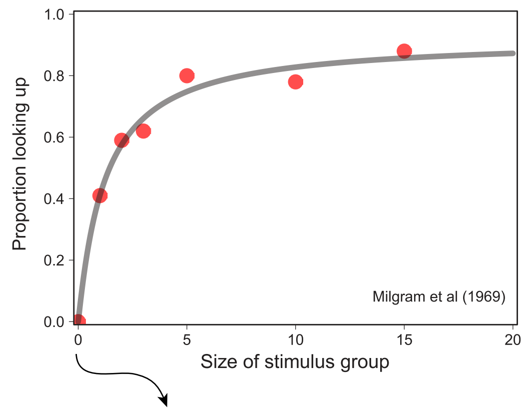


Herman Ebbinghaus(1850-1909)




## YAME (Yet Another Milgram Experiment)

As we shall see, deciding about the gain function is much trickier than the memory kernel. Milgram's crowd-gaze experiment provides an initial point for us to start asking questions. The experiment suggests a rapid build up of the crowd attention after being exposed to a small biased population. However, I believe this model is far too simplistic to express some of the peculiarities of humans' behavior in social nets.

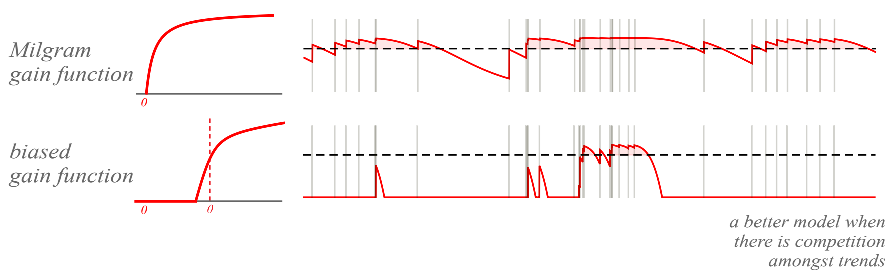




Stanley Milgram(1933-1984)

## Which Gain Function ?

The main problem with Milgram's gain function is that it almost passes everything! Functionally it means, the user's attention can be drawn by a very small stimulus. As there is no inhibition behavior in our model, such a gain function when considered in a feedback loop, can amplify any random information to become a persisting meme. This model can only be true when there is no competition amongst the trends. For now, considering a static shift can fix it and give more likely results.




## F=ma

Here we provide a simple stochastic model for the social intervention or the stimulus. It basically is a random spiking process over the network that is parameterized by two variables **m** (mass) and **a** (acceleration).

$$S_i(t,\mathbf{m},\mathbf{a}) = \delta(t - t_s - \xi^i_{\mathbf{a}}(t_s)) \times B_{\mathbf{m}}$$

$t_s$ *stimulation time*

*acceleration (synchronicity)* $\xi^i_{\mathbf{a}}(t_s) \sim \mathcal{N}(\mu : t_s, \sigma : \frac{\gamma}{\mathbf{a}})$ $\gamma > 0$ *just a constant*

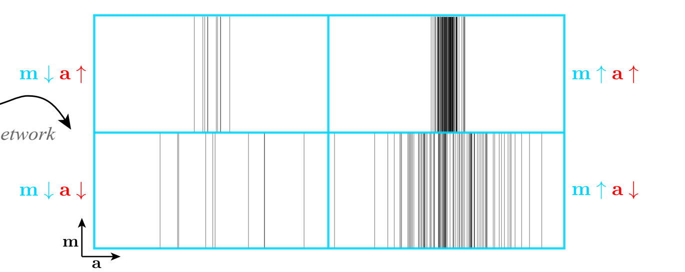
*mass (stimulated population)* $B_{\mathbf{m}} \sim \mathrm{Ber}(\frac{\mathbf{m}}{N})$ *Bernoulli Distribution with p=m/N*

This figure shows the stochastic process of sum of interventions over the network for four processes generated with high and low values of **m** and **a**.

As we see, a *high value for mass means more people are targeted* and *high value for acceleration means they are targeted more synchronously*.

$$\sum_{i=1}^{N} S_i(t,\mathbf{m},\mathbf{a})$$

*stimulation over the network*




Now by defining the social force similar to that of RBNNs, as *F*, we can write the social influence dynamics equation where the effect of social intervention (e.g. Media) is a function mass and acceleration.

$$\tau_i \frac{\mathrm{d}F_i(t)}{\mathrm{d}t} = -F_i(t) + \sum_{j=1}^{N} A_{ij}\tilde{g}(F_j(t)) + S_i(t,m,a)$$

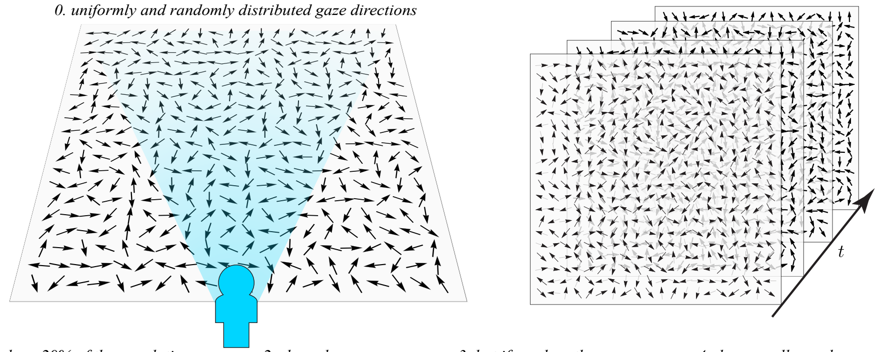
 

## An Enhanced and Inexpensive YAME

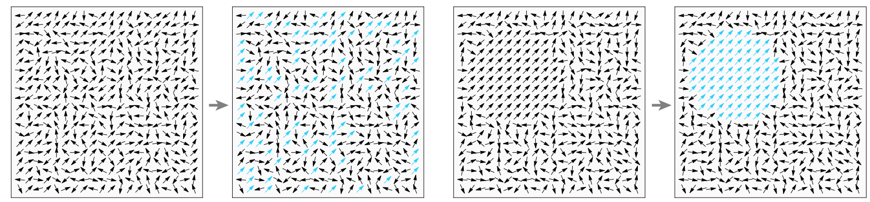
Let us consider a room paved by arrows indicating the gaze direction of people in YAME(left figure). To be more accurate, we could take a space-time volume of the arrows (right figure) to study the temporal density of events but we will see this simplification won't hurt much. Now we can set the crowd's gaze as we wish and see what draws our attention. To begin with, let us re-examine what we already know from YAME on the effect of *mass* and see how this visualization can demonstrate the effect of *acceleration* as well.

In the figure 1 , we have skewed a remarkable portion of the population from the randomly distributed version in the figure 0 but it is actually hard to notice without highlighting (figure 2). On the other hand, figure 3 and 4 show the effect of focusing the same degree of intervention in a compact area. It is worth mentioning that spatial focus in our visualization is analogous to temporal focus (synchronicity) in our definition of acceleration.

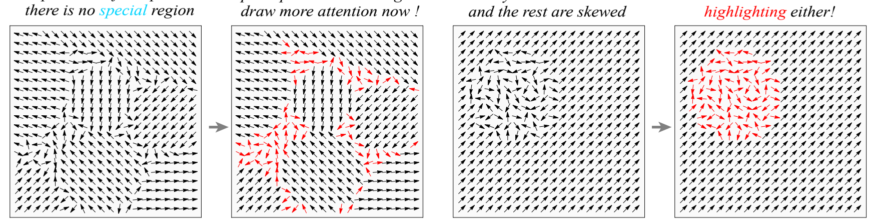
*0. uniformly and randomly distributed gaze directions*




*1. here 20% of the population are randomly chosen and skewed* — *2. skewed arrows are highlighted* — *3. but if we skew the same percentage in a compact region:* — *4. do we really need this highlighting?*



*5. in presence of competition, there is no special region* — *6. perhaps the random regions draw more attention now !* — *7. only 20% are random and the rest are skewed* — *8. we don't need this highlighting either!*



## It's Complicated

Now let us consider the effect of competing trends that is neglected in Milgram's model. Figure *5* and *6* demonstrate the effect of presence of too many gaze directions with high mass and acceleration. The competition breaks the *F=MA* law and suddenly there is no special region to draw our attention.

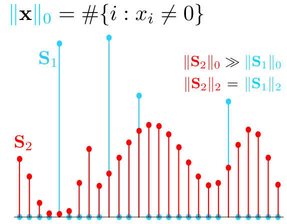
The other interesting phenomenon occurs when we increase the population of skewed arrows. As they become the majority, the still-random portion gets more attention. We call this phenomena *Attention Inversion* (figure 7 and 8).

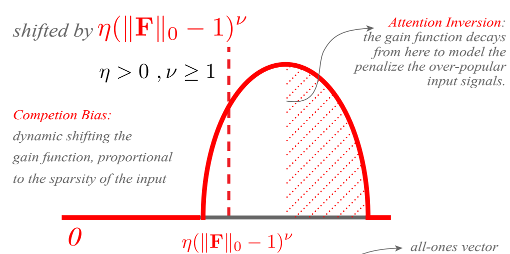
Examples of these phenomena can be seen in Twitter and Facebook. For instance, new Facebook users generally tend to share contents more frequently than the older ones. The reasons are they have less connections, Liked pages and groups and do not perceive the competition between trends as much as more connected users do. As an example for the Attention Inversion phenomenon, when a trending topic in social networks gets too much popularity, sharing or re-tweeting is not interesting anymore as many people in the network has already done it.

## The New Model

Here we put our observations from the last section in a mathematical framework and construct a better model. We consider our signals are no longer 1D spikes. They live in a high-dimensional space where each dimension stands for a different topic in the network and we call it the *Topic-Space*. To consider the *competition bias*, signals with less number of topics (sparse signals) are more impactful than those with distributed energy over all dimensions. To measure the number of nonzero elements in a vector we use the zero-norm which is not really a norm though! In the left figure, the blue and red signals have the same energy (two-norm) but the blue signal is much sparser (low zero-norm). Using this measure we can shift the gain function dynamically according to the sparsity of the input(right figure). This makes the gain function more permeable to sparser signals.

$\|\mathbf{x}\|_0 = \#\{i : x_i \neq 0\}$



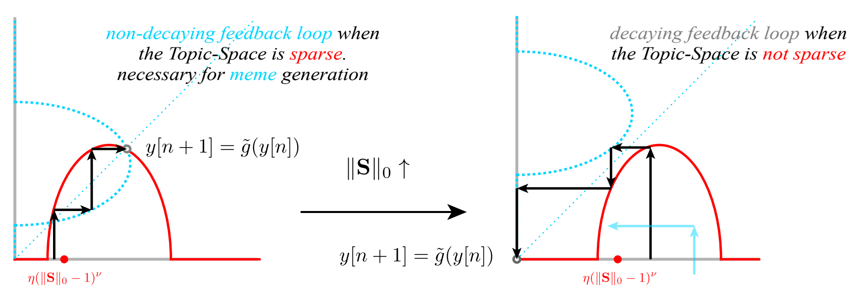

all-ones vector

$$\tau_i \frac{\mathrm{d}\mathbf{F}_i(t)}{\mathrm{d}t} = -\mathbf{F}_i(t) + \sum_{j=1}^{N} A_{ij}\tilde{g}(\mathbf{F}_j(t) - \eta\mathbb{I}(\|\mathbf{F}_j\|_0 - 1)^\nu) + \mathbf{S}_i(t) - \eta\mathbb{I}(\|\mathbf{S}_i\|_0 - 1)^\nu$$

The easiest way to model the *attention inversion* phenomenon is to choose a *compact-support* gain function (right figure). In other words, the gain function should decay after a certain point to surpass the *over-popular* signals and stop them from drawing attention. The equation above shows the modified model with dynamically shifting gain function.

Now this model should be capable of expressing some of the complex behaviors of social networks. For example, here we see the effect of sparsity when the gain function is put in a positive feedback loop.

In our future work, we will perform Monte Carlo simulation of this model to assess its correctness and predict/study more complex behaviors. Thanks for reading!




## References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**Milgram, S., Bickman, L., & Berkowitz, L. (1969).** Note on the drawing power of crowds of different size. Journal of personality and social psychology, 13(2), 79.


 